

Neutrino mass hierarchy and neutrino oscillation parameters with 100,000 reactor events

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see the paper for references and details

Plan of the talk

Introduction

Theoretical rate calculation: cross section and oscillation probability

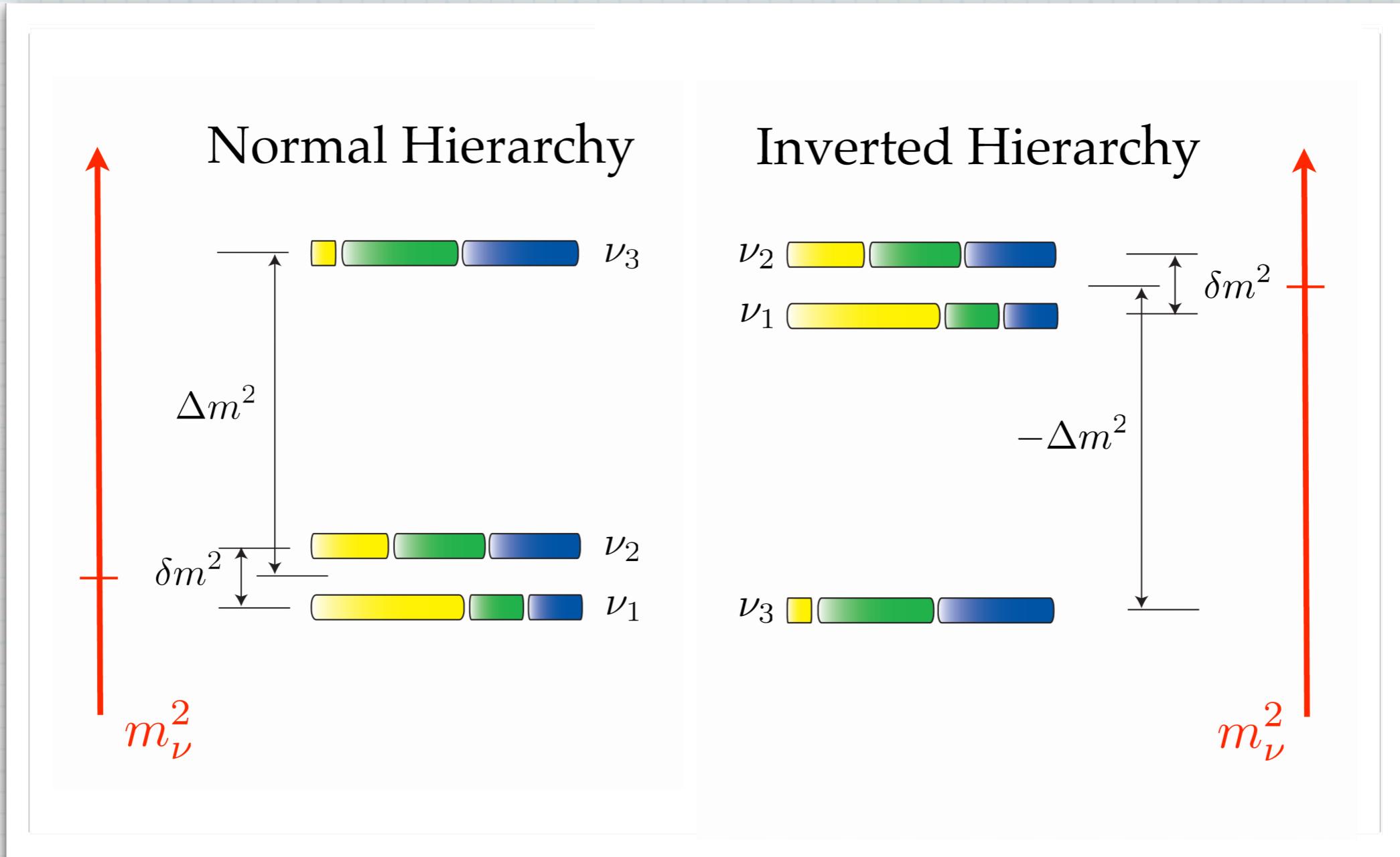
Continuous parameter α to interpolate between hierarchies

Results of the analysis of prospective data:
hierarchy and oscillation parameters

Possible impact of energy scale errors and spectral shape uncertainties

Conclusions

Notation



$$(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$$

$$c_{ij} = \cos \theta_{ij}, \quad s_{ij} = \sin \theta_{ij}$$

$$\alpha = \begin{cases} +1 & (\text{normal hierarchy}) \\ -1 & (\text{inverted hierarchy}) \end{cases}$$

Medium-Baseline Reactor Neutrino Experiment (RENO-50, JUNO)

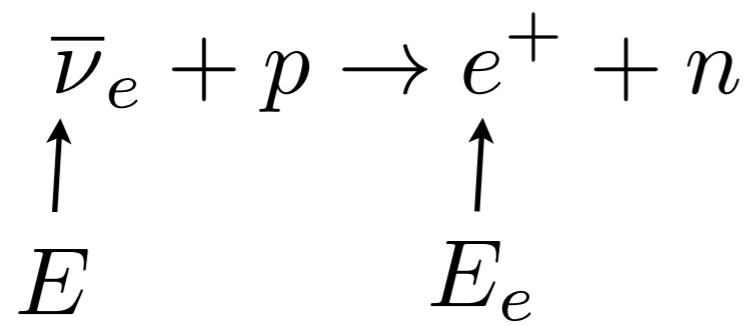
Possible discrimination of the hierarchy via high-statistics reactor neutrino experiments at medium baselines (few tens of km) was proposed more than 10 years ago

Probe mass-mixing parameters which govern oscillations at short wavelength ($\delta m^2, \theta_{12}$) and at long wavelength ($\Delta m^2, \theta_{13}$), and their tiny interference effects which depend on the mass hierarchy

Require unprecedented levels of detector performance and collected statistics, and the control of several systematics at (sub)percent level

Therefore, accurate theoretical calculations of reactor event spectra and refined statistical analyses are needed

Inverse Beta Decay cross section



In our rate calculation we use the differential cross section

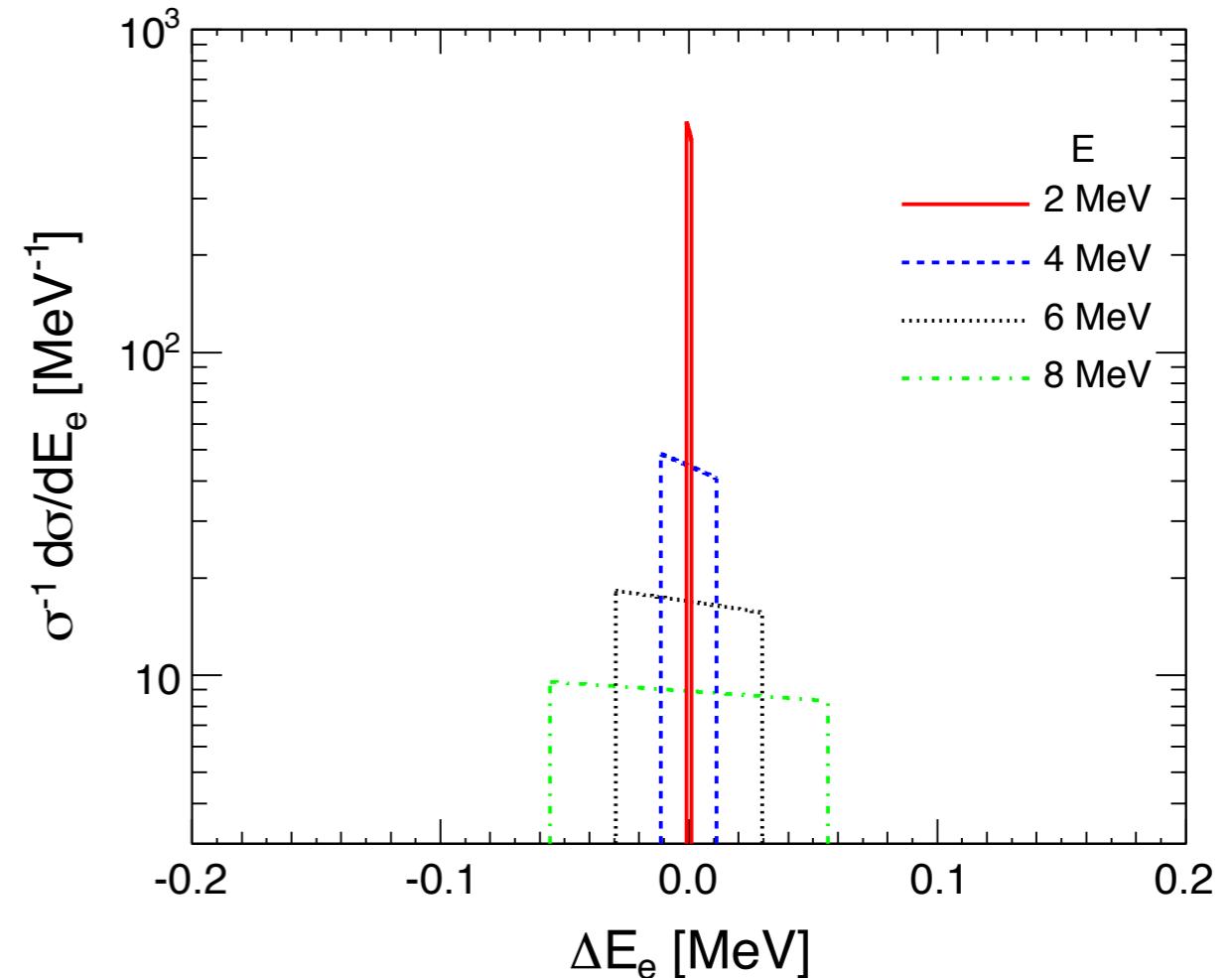
$$d\sigma(E, E_e) / dE_e$$

$O(E/m_p)$ correction to the relation

$$E - E_e = m_n - m_p = 1.293 \text{ MeV}$$

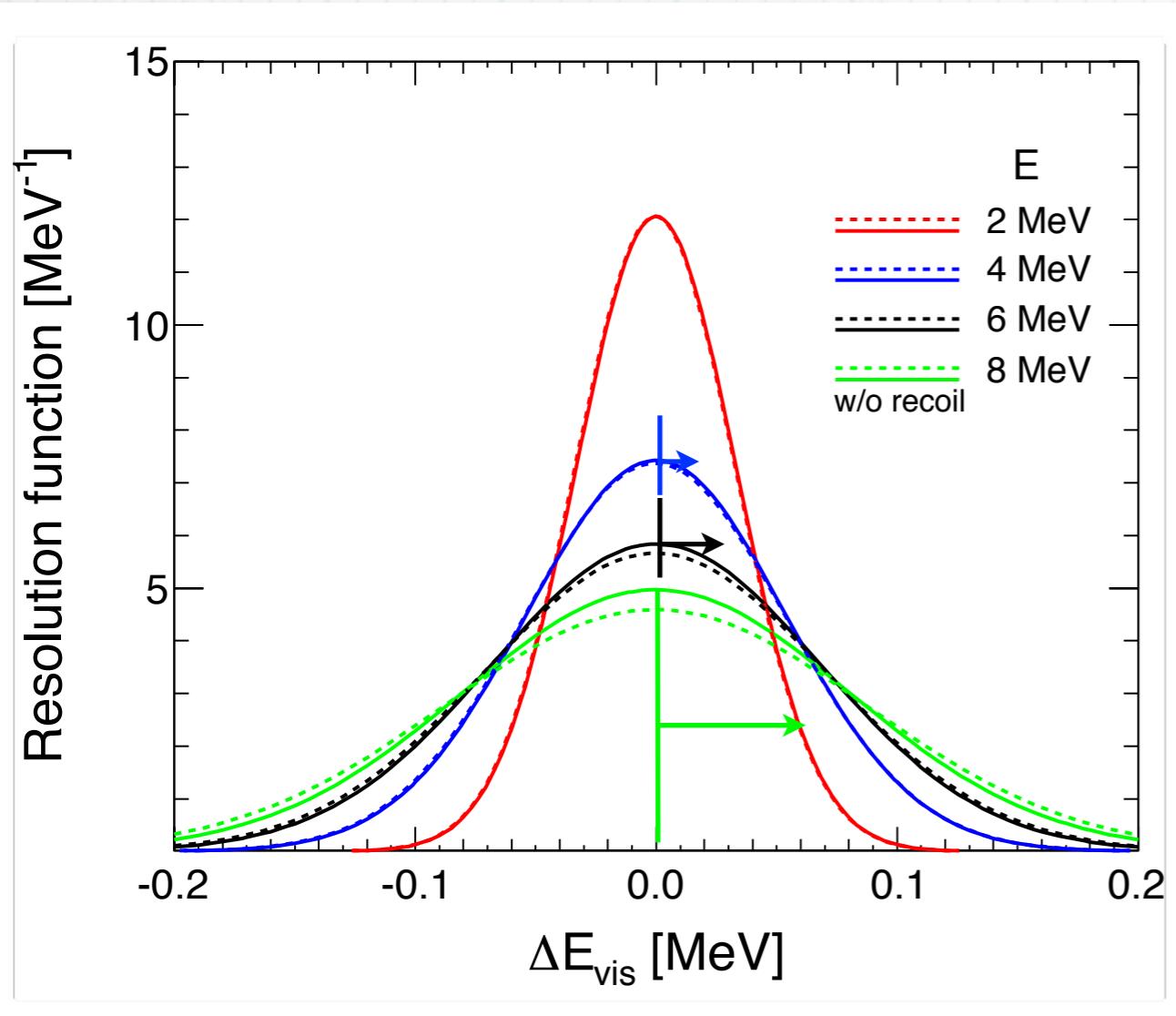
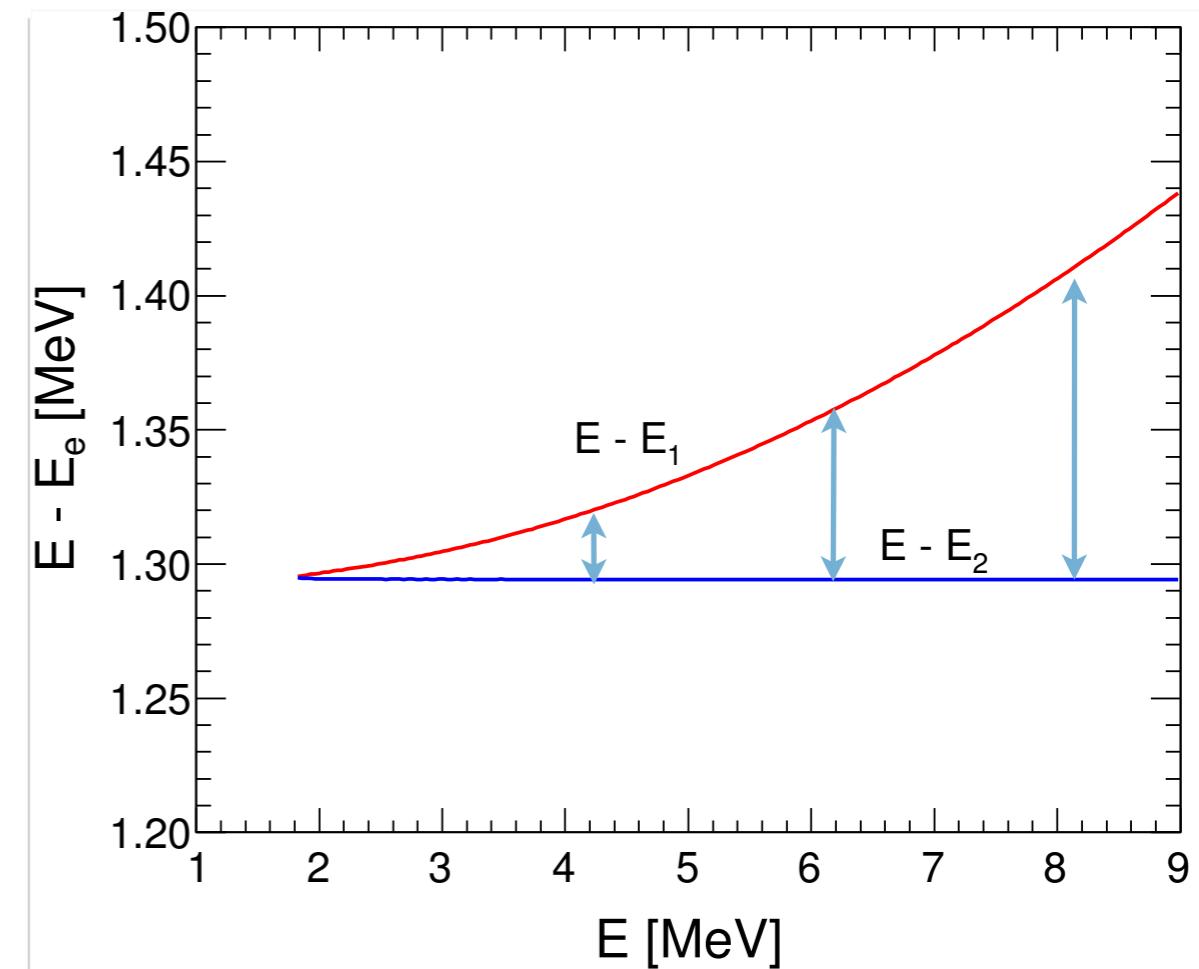
with respect to recoilless approximation

Two effects
 Tipical E_e displaced by $O(E/m_p)$
 E_e spread of order $O(E/m_p)$



In the high energy part of the spectrum effect same order of energy resolution, cannot be neglected

Boundaries of the neutrino-positron energy difference $E - E_e$



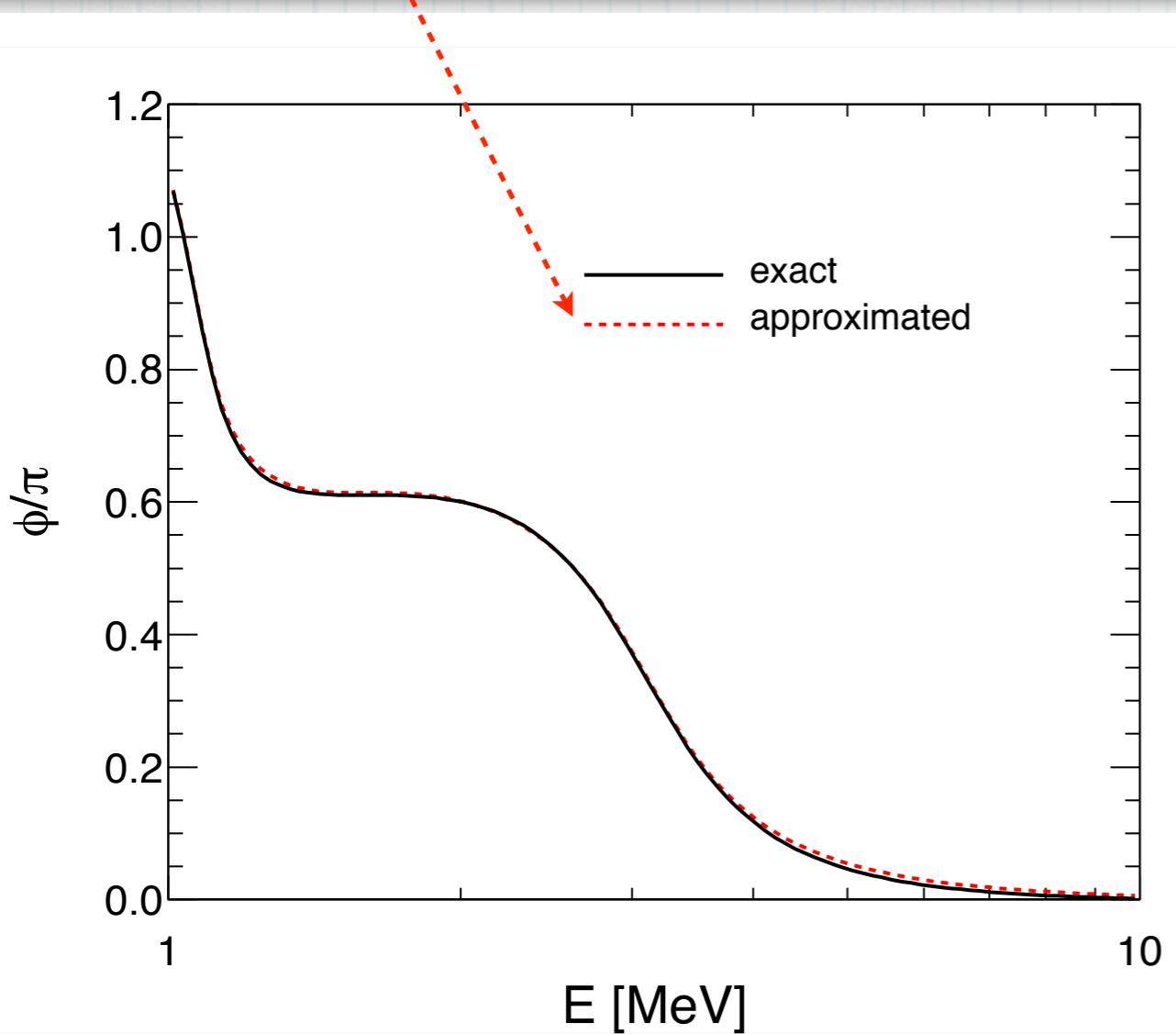
Recoil-corrected energy resolution function (functions aligned to their median value)

Oscillation probability

$$P_{\text{vac}}^{3\nu} = c_{13}^4 P_{\text{vac}}^{2\nu} + s_{13}^4 2s_{13}^2 c_{13}^2 \sqrt{P_{\text{vac}}^{2\nu}} \cos(2\Delta_{ee} + \alpha\varphi)$$

$$\varphi \simeq 2s_{12}^2 \delta \left(1 - \frac{\sin \delta}{2\delta \sqrt{P_{\text{vac}}^{2\nu}}} \right)$$

$$\alpha = \begin{cases} +1 & (\text{normal hierarchy}) \\ -1 & (\text{inverted hierarchy}) \end{cases}$$



continuous parameter constrained
by the fit statistically appropriate
for estimation test

hierarchy determination
compromised if $\alpha \sim 0$
allowed or preferred

Oscillation probability: matter effects, multiple reactors

Analytical approximation for the oscillation probability in matter and for multiple reactors

$$P_{\text{mat}}^{3\nu} \simeq c_{13}^4 P_{\text{mat}}^{2\nu} + s_{13}^4 + 2s_{13}^2 c_{13}^2 \sqrt{P_{\text{mat}}^{2\nu}} w \cos(\Delta_{ee} + \alpha\varphi)$$



The fractional matter correction to $(\theta_{12}, \delta m^2)$ is $\sim 8 \times 10^{-3}$ in the high energy part of the spectrum, of the same size of the prospective fit accuracy

Replace $(\theta_{12}, \delta m^2) \rightarrow (\tilde{\theta}_{12}, \delta \tilde{m}^2)$ in $P_{2\nu}^{\text{vac}}$

$w \longrightarrow$ Damping factor (analytical) to account for multiple reactors

Differences with the exact probability can be safely neglected in the analysis ($\lesssim 2 \times 10^{-3}$)

We consider the JUNO experimental settings

10 Reactor Cores $L \sim 52.5$ km $P = 35.8$ GW

Detector mass $M = 20$ kT

5 year exposure

$3\% \sqrt{(m_e + E_e)/\text{MeV}}$ energy resolution

With these assumptions

3.4×10^5 unoscillated events
 $\sim 10^5$ events with oscillations

2 Far reactors

$$L_1 = 215 \text{ km}$$

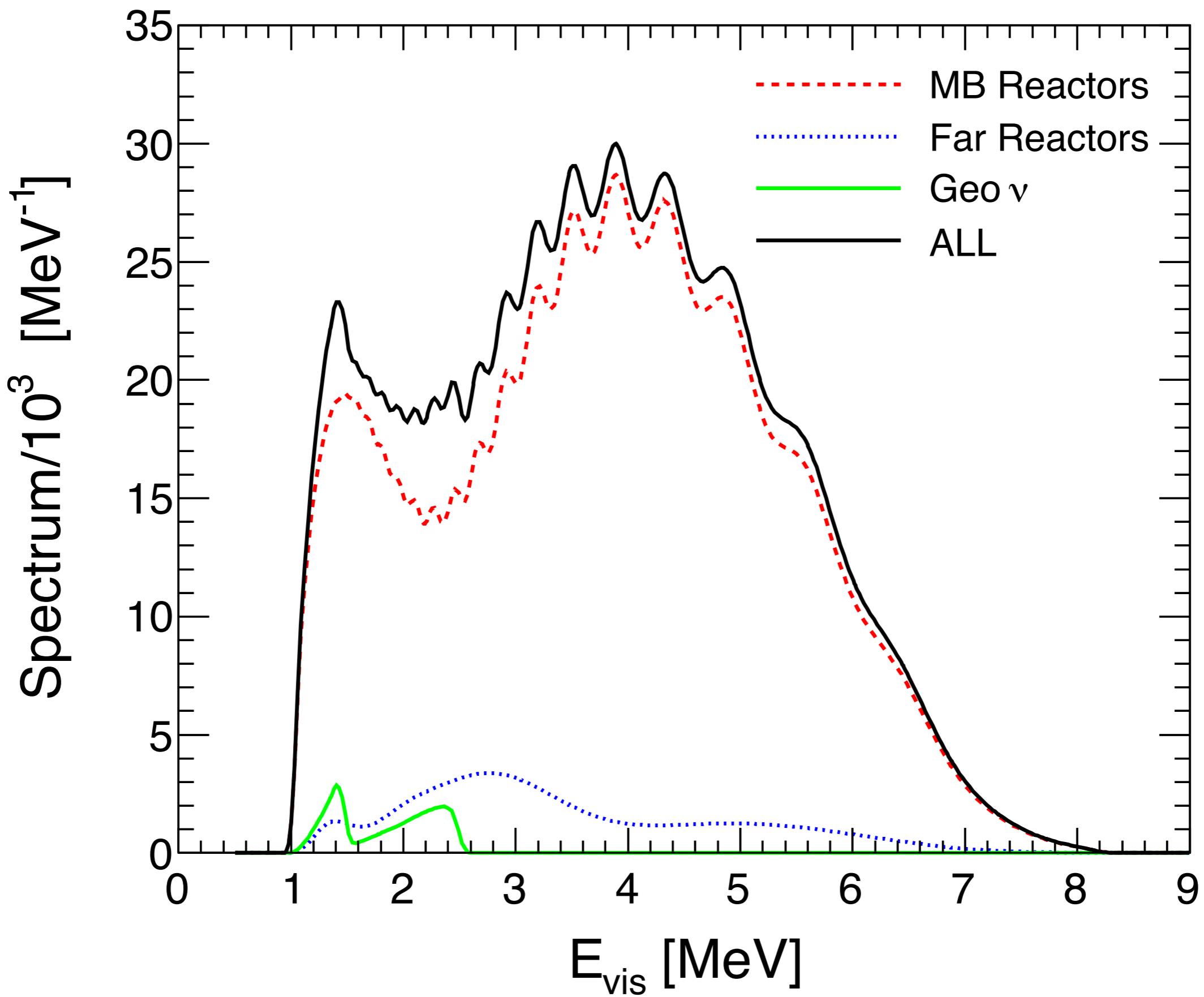
$$L_2 = 264 \text{ km}$$

$$P = 17.4 \text{ GW}$$

unoscillated events
 $\sim 1.65 \times 10^4$

Geoneutrinos (mostly from Uranium and Thorium)

unoscillated events
 3.5×10^3



DATA ANALYSIS

True spectrum calculated for global analysis best-fit values
of oscillation parameters and fixed NH (IH) hierarchy

$$\chi^2 = \chi_{\text{stat}}^2 + \chi_{\text{par}}^2 + \chi_{\text{sys}}^2$$

$$\chi^2 = \chi^2(\delta m^2, \Delta m_{ee}^2, \theta_{12}, \theta_{13}, \alpha, f_R, f_U, f_{\text{Th}}) \quad (\text{limit of infinite bins})$$

$$\chi_{\text{par}}^2 = \sum_{i=1}^4 \left(\frac{p_i - \bar{p}_i}{\sigma_i} \right)^2$$

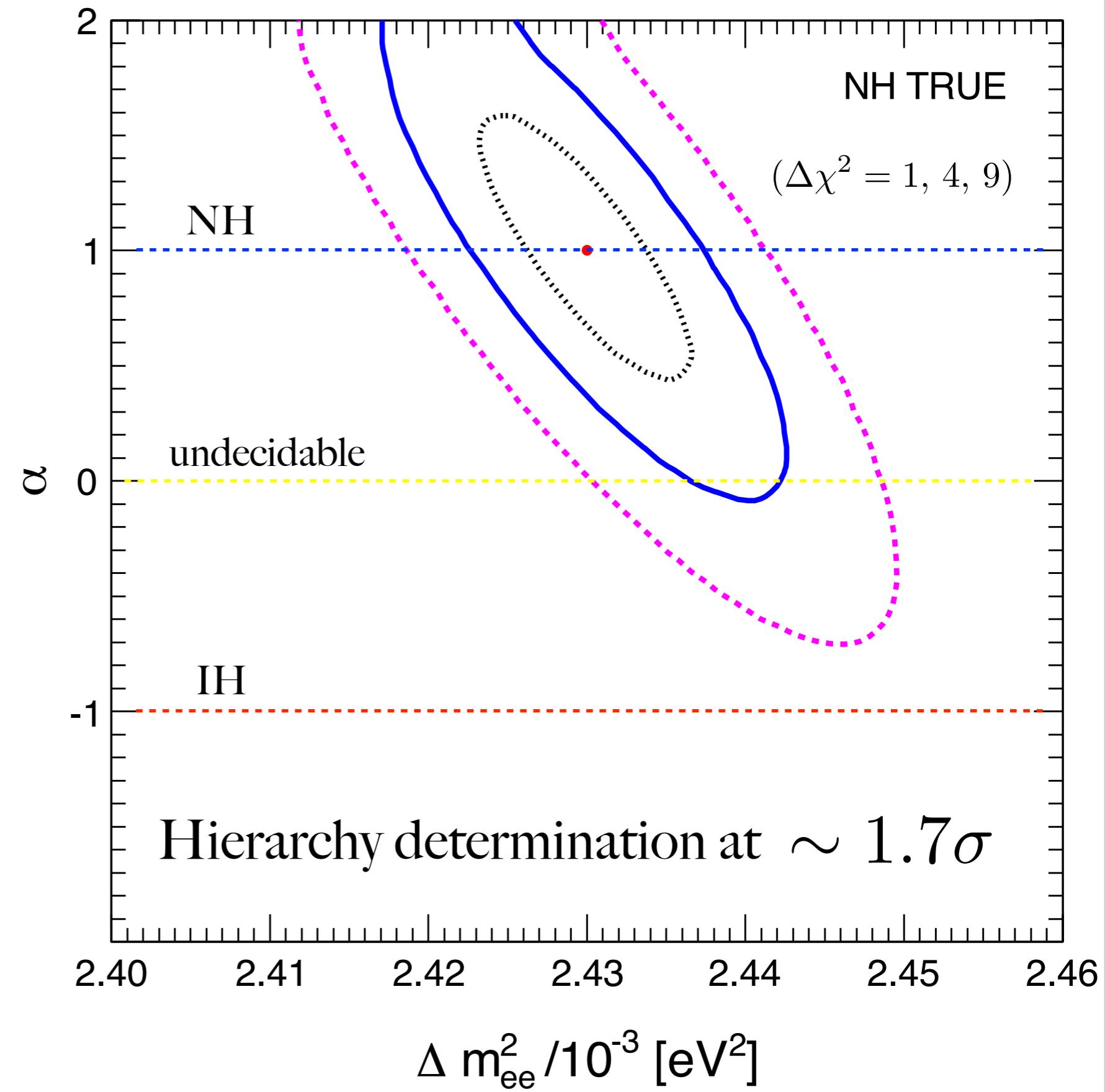
$$\chi_{\text{sys}}^2 = \sum_{j=R,U,\text{Th}} \left(\frac{f_j - 1}{s_j} \right)^2$$

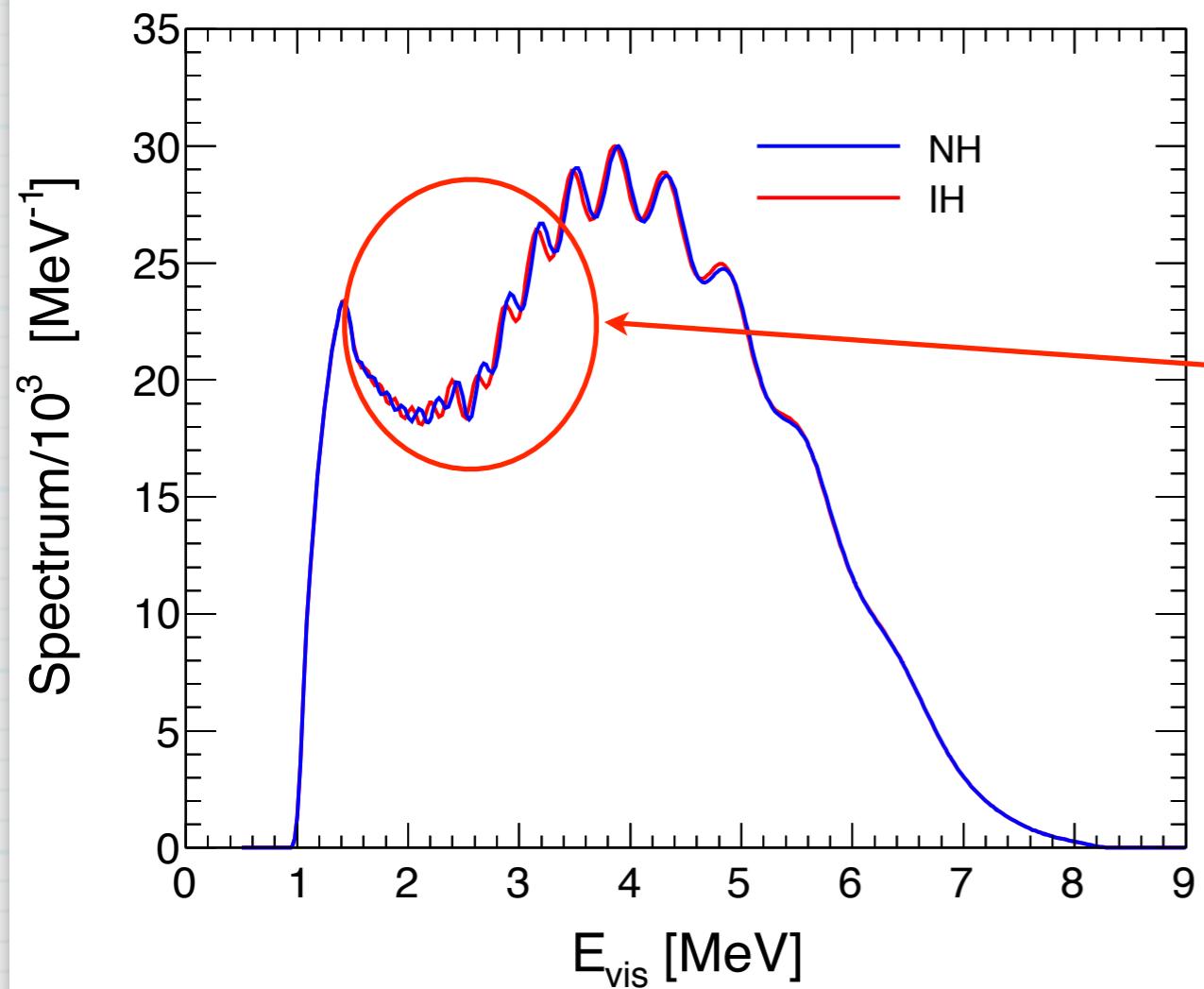
Parameter	% error		% error after fit (NH true)			% after fit (IH true)		
	(prior)	all data	all – far	all – geo	all data	all – far	all – geo	
α	∞	59.2	59.0	57.0	56.2	55.3	54.0	
Δm_{ee}^2	2.0	0.26	0.25	0.26	0.26	0.25	0.25	
δm^2	3.2	0.22	0.21	0.16	0.21	0.21	0.16	
s_{12}^2	5.5	0.49	0.47	0.39	0.49	0.46	0.42	
s_{13}^2	10.3	6.95	6.88	6.95	6.84	6.77	6.84	
f_R	3.0	0.66	0.66	0.64	0.65	0.65	0.64	
f_{Th}	20.0	15.3	14.6	—	15.5	15.4	—	
f_U	20.0	13.3	13.3	—	13.3	13.3	—	

Distance between NH
and IH ($\alpha = \pm 1$)
at $\sim 3.4\sigma$

We recover the approximate
“factor of two” reduction of
the sensitivity with respect
to naive expectations

$$N_\sigma \simeq 0.5\sqrt{\Delta\chi^2(\text{NH} - \text{IH})}$$

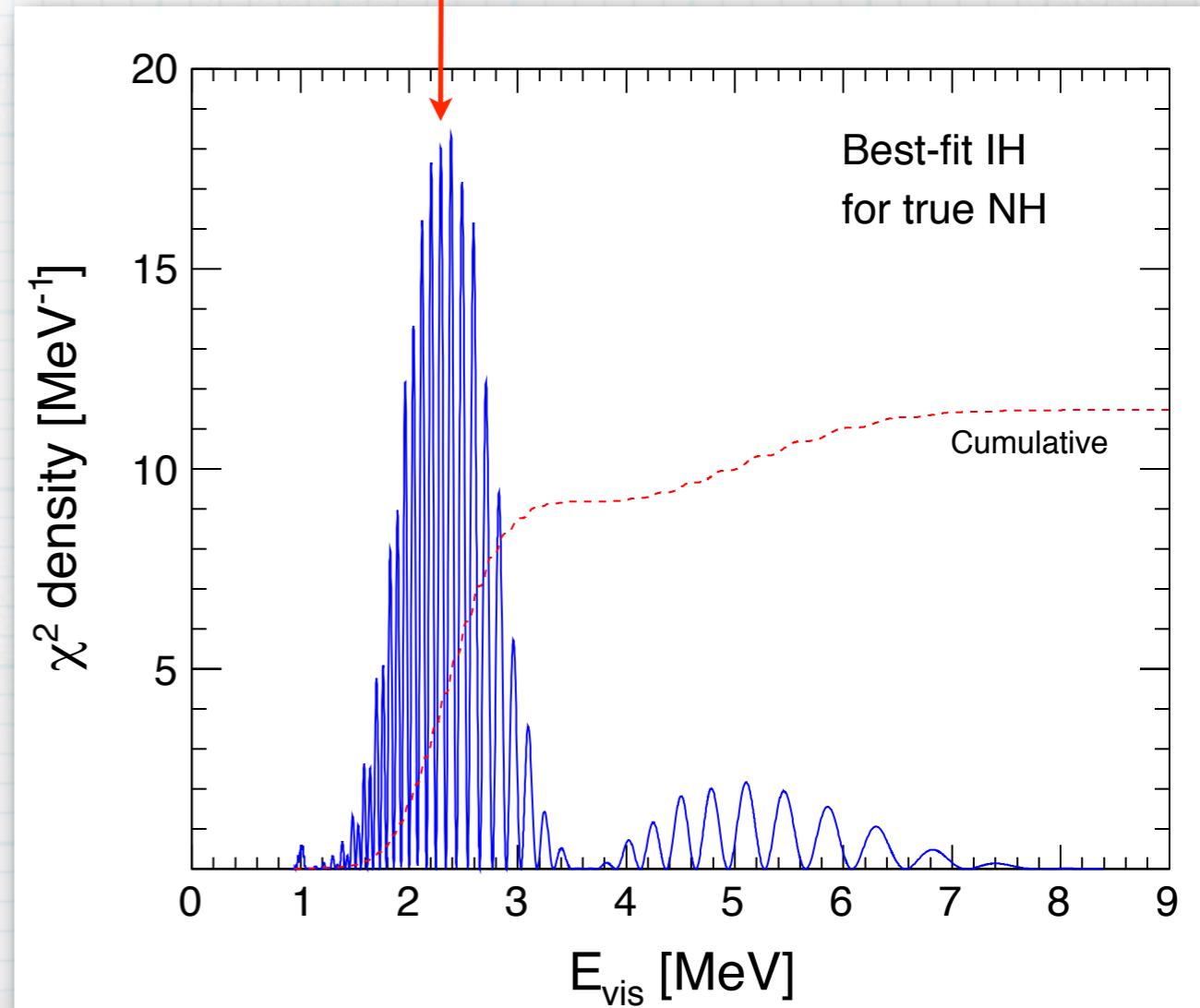




Comparison between NH reference data point and best fit for IH

Comparison between NH and IH, at central value oscillation parameters

Most of the discriminating power in the energy range $2 - 3 \text{ MeV}$



Energy Scale Error: possible nonlinear $E \rightarrow E'$ transformation

Hierarchy degeneracy
 $\alpha = \pm 1 \rightarrow \alpha = \mp 1$
 if $E \rightarrow E'$ satisfies

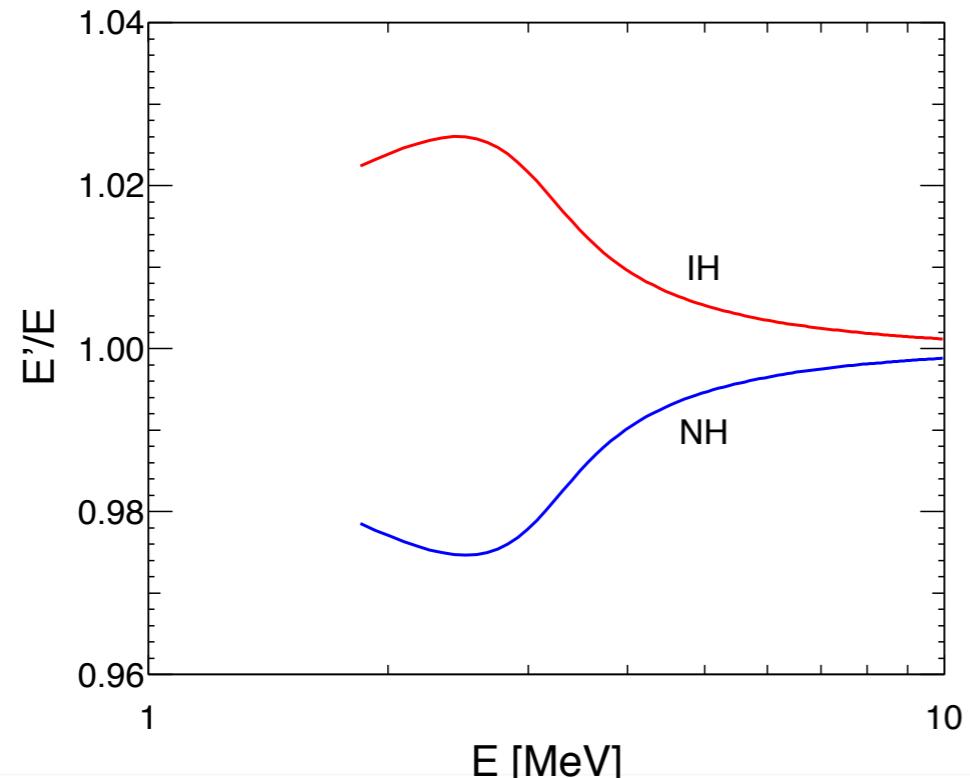
$$\frac{\Delta m_{ee}^2 L}{2E} \pm \varphi(E) = \frac{\Delta m_{ee}'^2 L}{2E'} \mp \varphi(E')$$

Good approximation

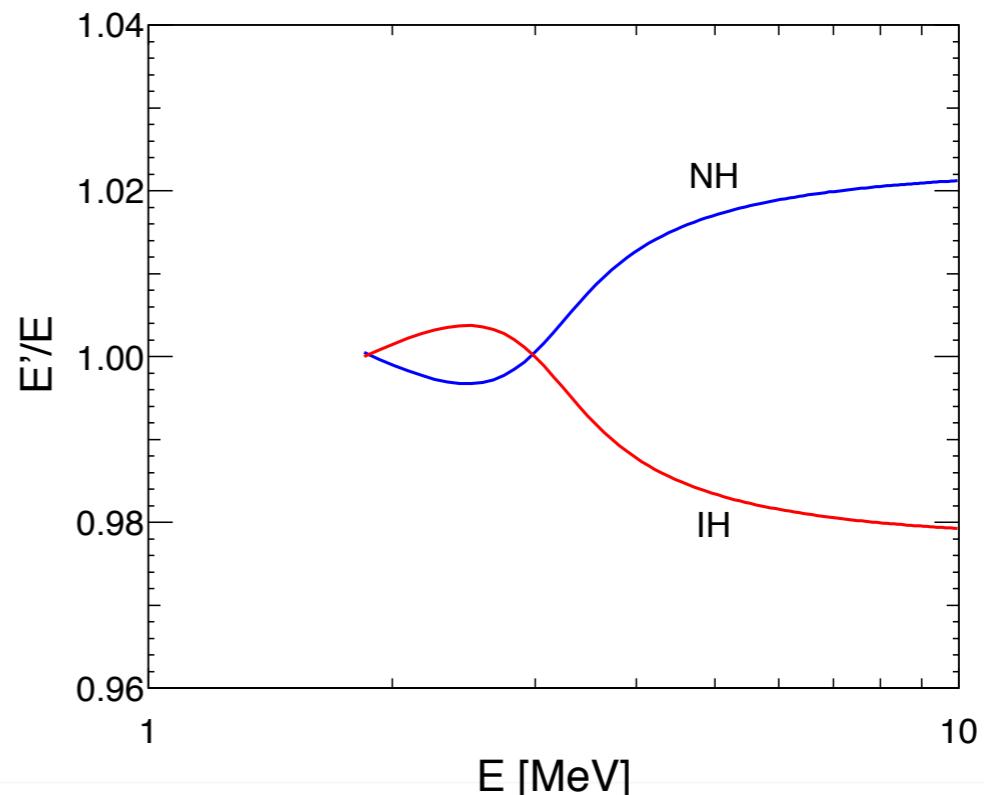
$$\frac{E'}{E} \simeq \frac{\Delta m_{ee}'^2}{\Delta m_{ee}^2} \mp 2s_{12}^2 \frac{\delta m^2}{\Delta m_{ee}^2} \left(1 - \frac{\sin \delta(E)}{2\delta(E)\sqrt{P_{\text{vac}}^{2\nu}(E)}} \right)$$

Two interesting cases

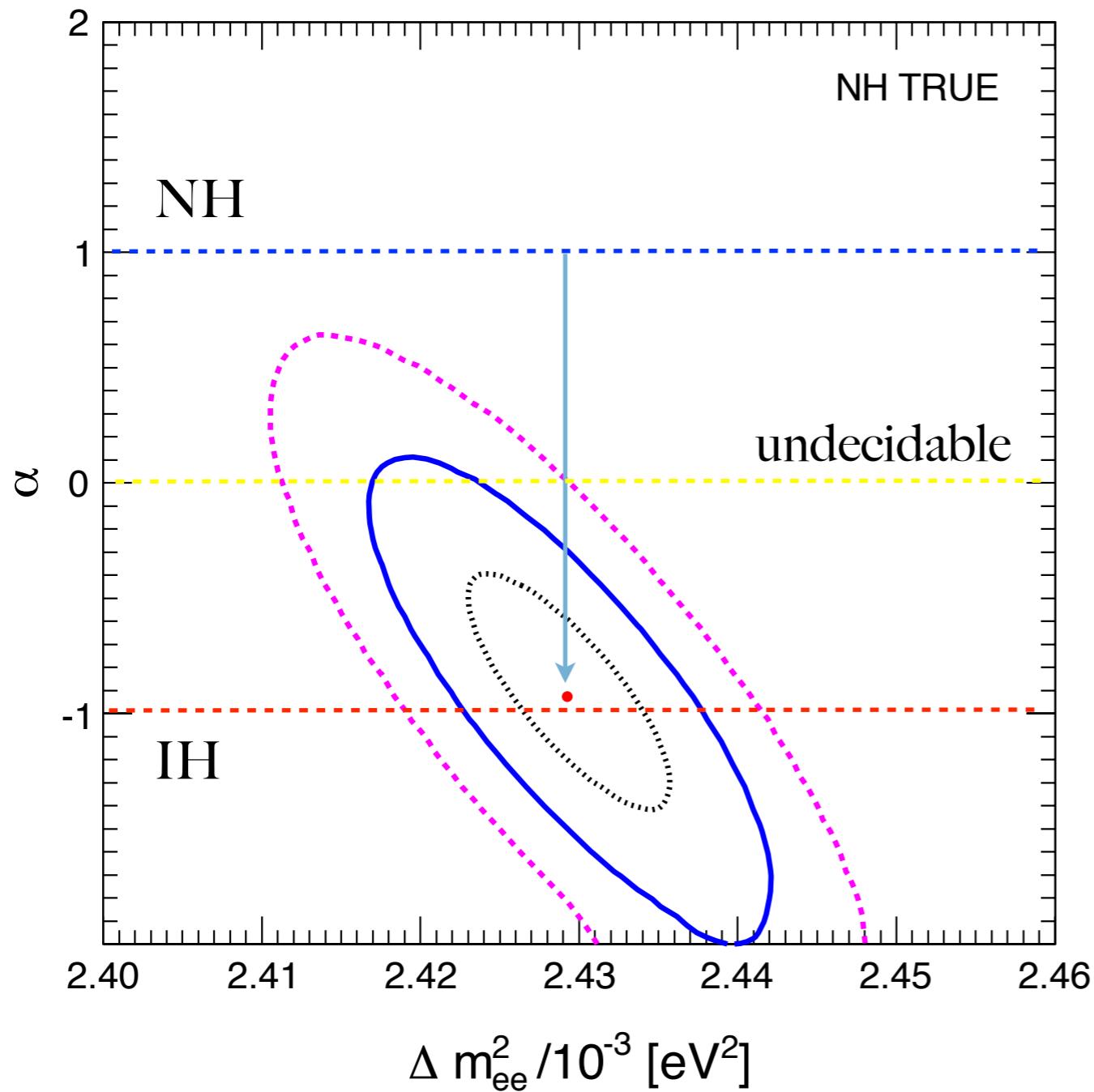
$(E \rightarrow E' \text{ with } E = E' \text{ at } \infty)$



$E \rightarrow E' \text{ with } E = E' \text{ at } E = E_T$



Analysis with energy scale error



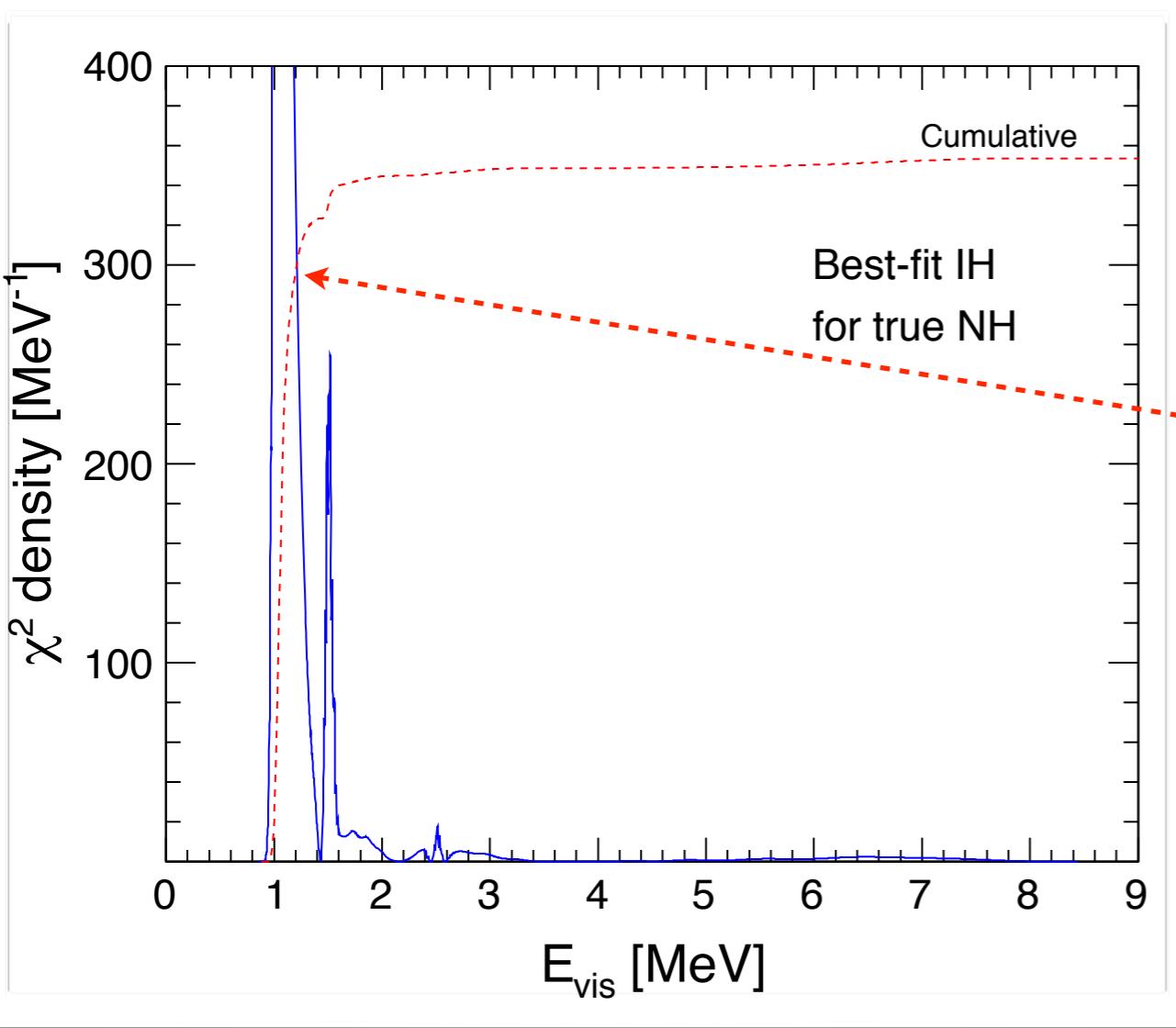
Assume NH true

Energy scale error
($E \rightarrow E'$ with $E = E'$ at ∞)

Hierarchy flipped
IH preferred

NH excluded at
more than 3σ

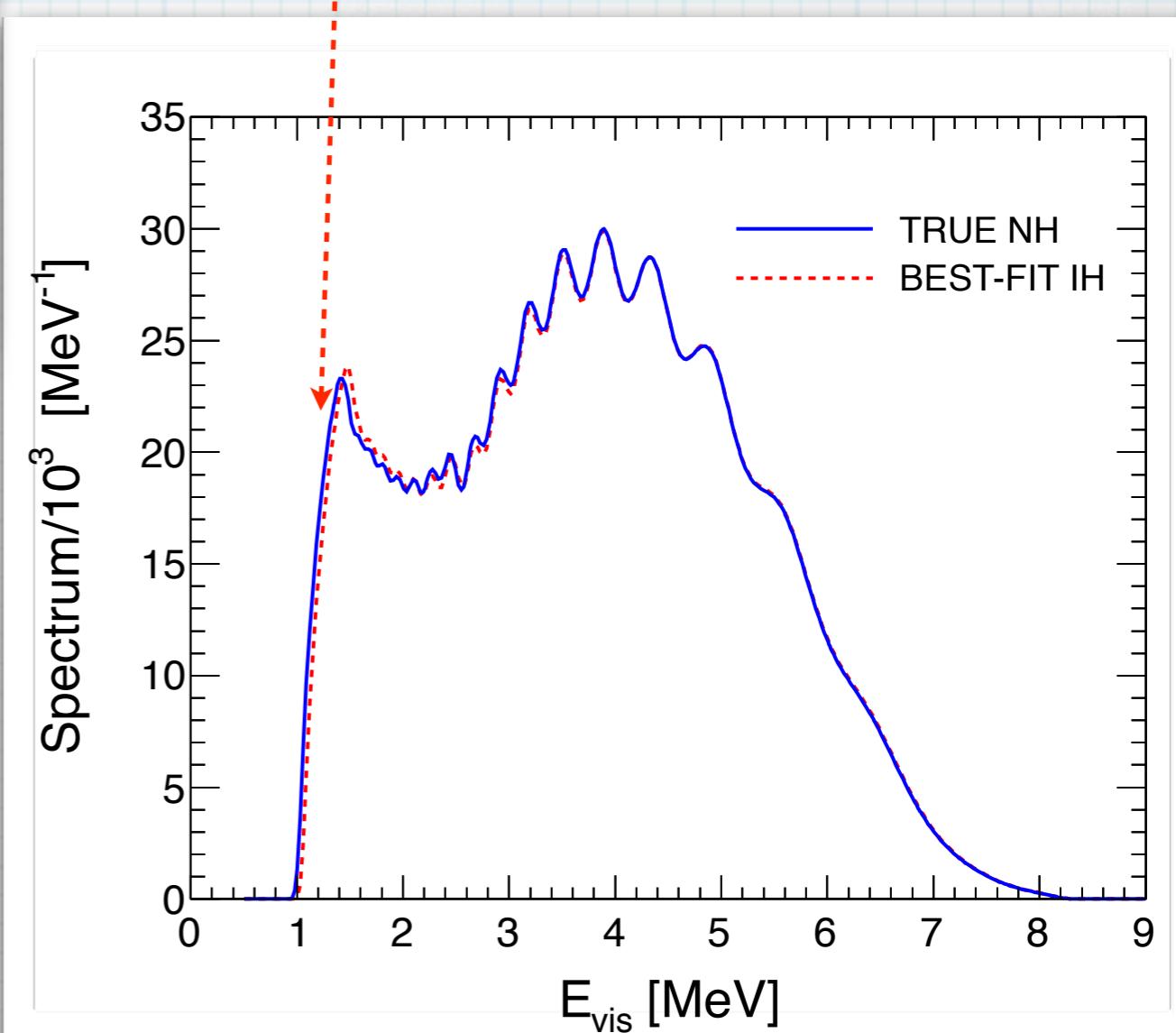
but ...

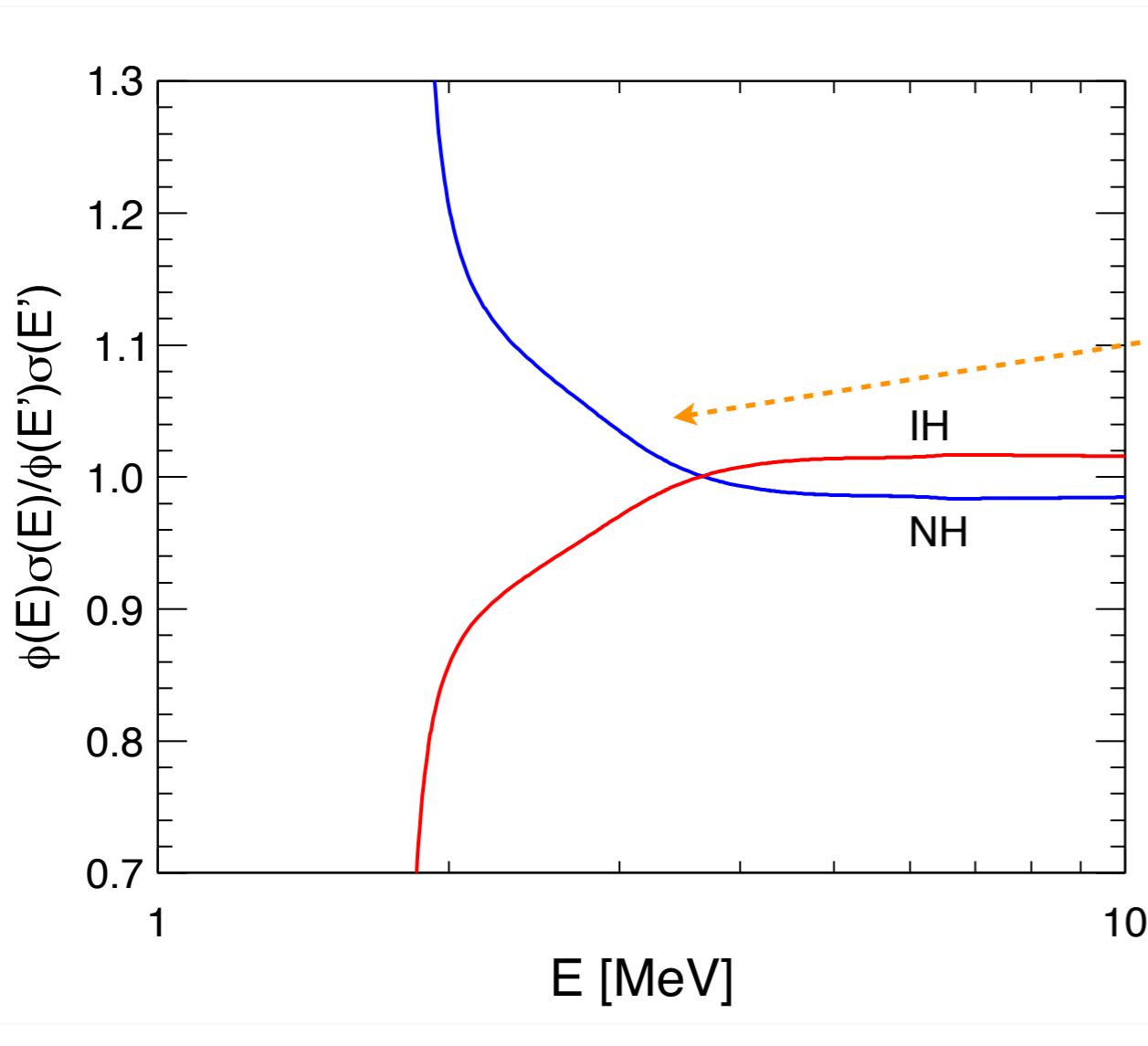


The low-energy tail of the observed spectrum may act as a self-calibrating tool to diagnose energy scale shifts at percent level near the threshold

Fit very poor $\chi^2 \sim 360$

Mismatch of spectra near the threshold and at geoneutrino spectral steps





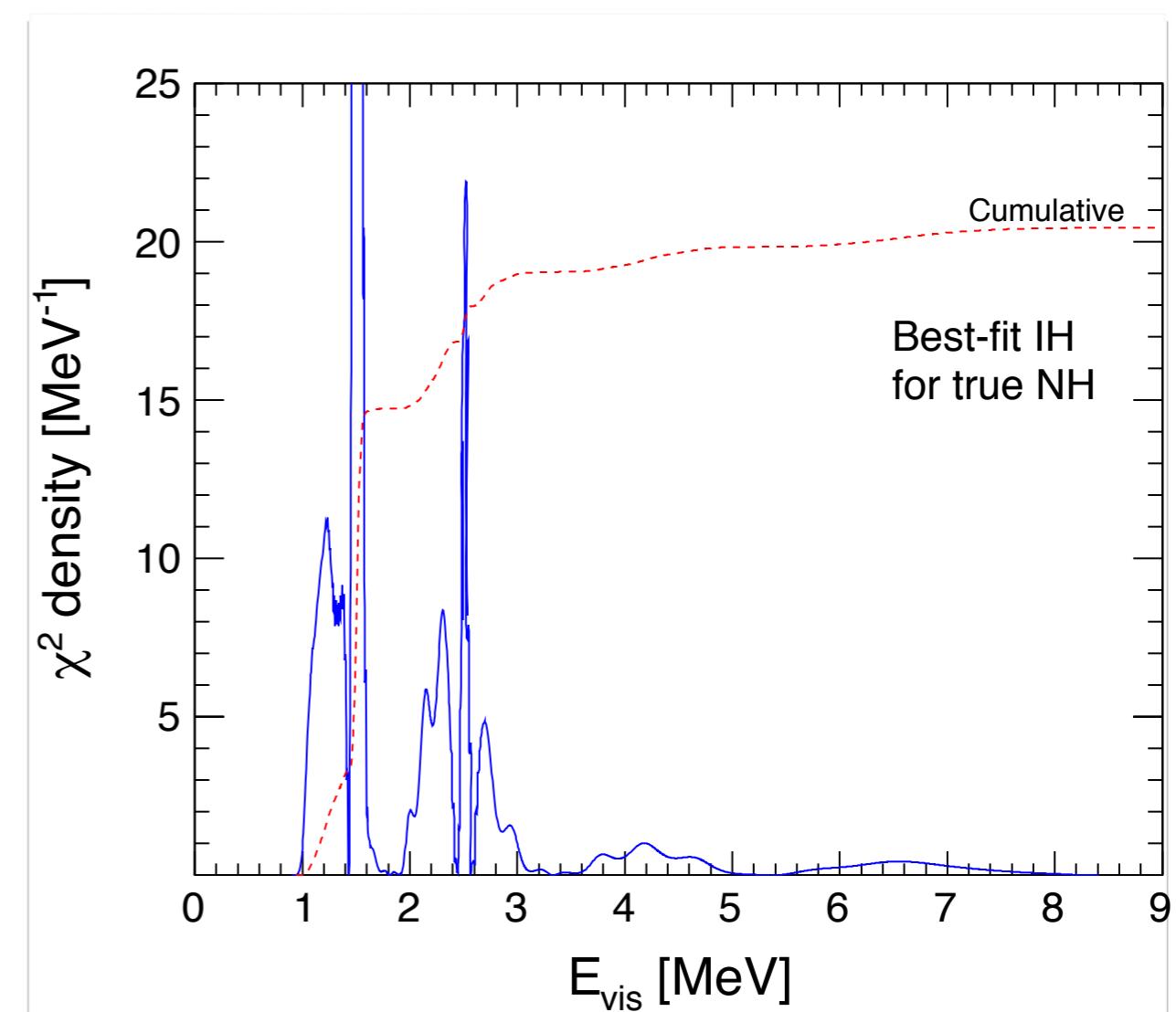
the degeneracy $\alpha = \pm 1 \rightarrow \alpha = \mp 1$
is then almost complete $\chi^2 \sim 22$

Analogue results for the case
 $E \rightarrow E'$ with $E = E'$ at $E = E_T$

If the shape errors in the observable reactor spectrum could **compensate** the change

$$\Phi(E)\sigma(E) \rightarrow \Phi(E')\sigma(E')$$

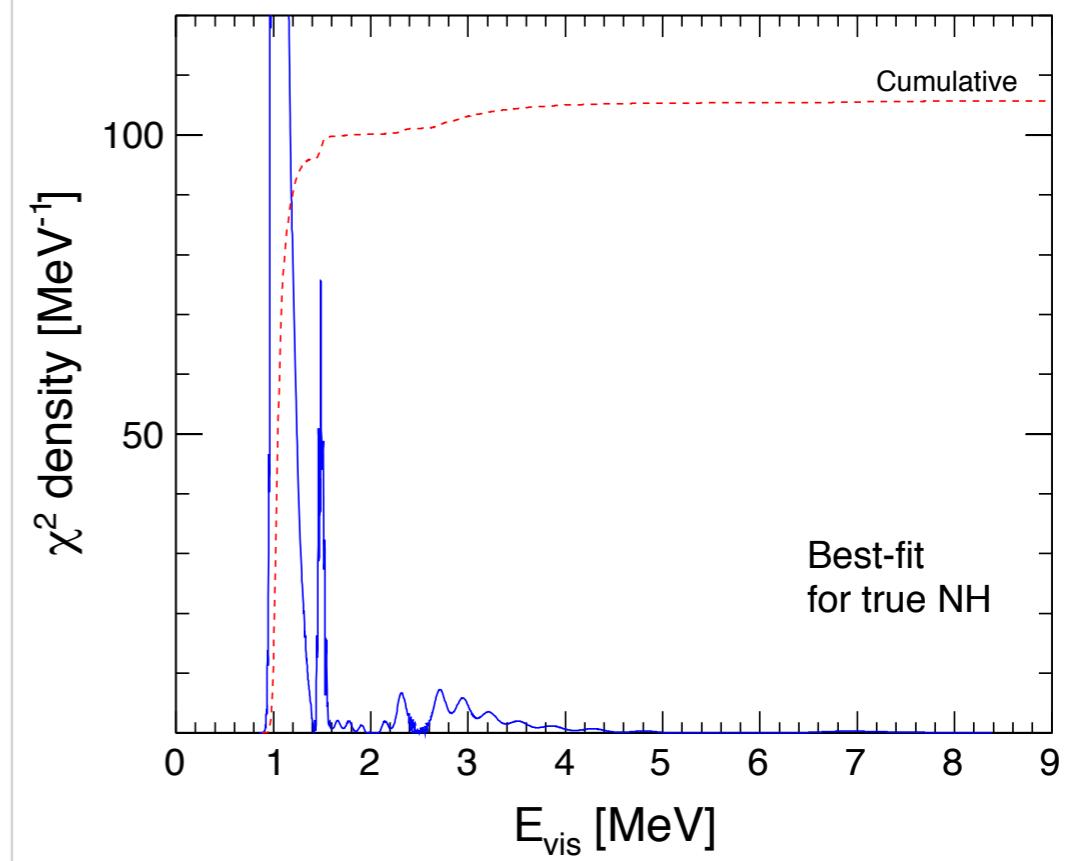
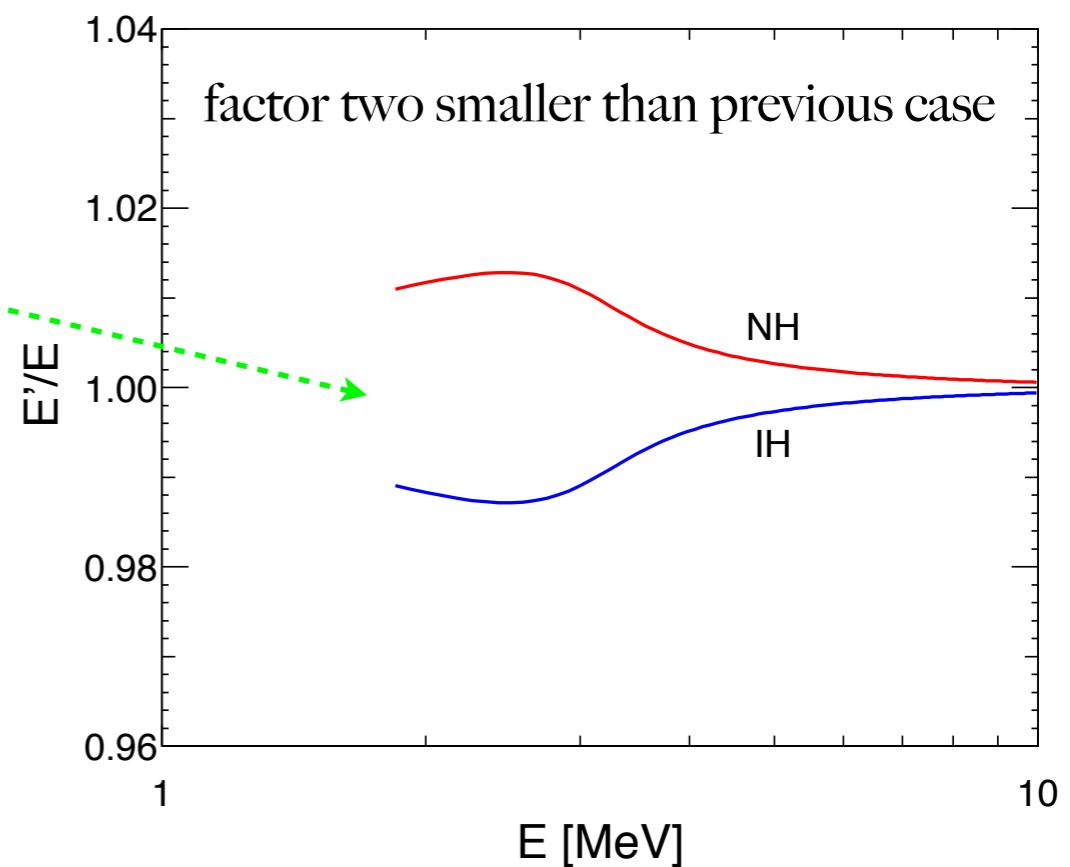
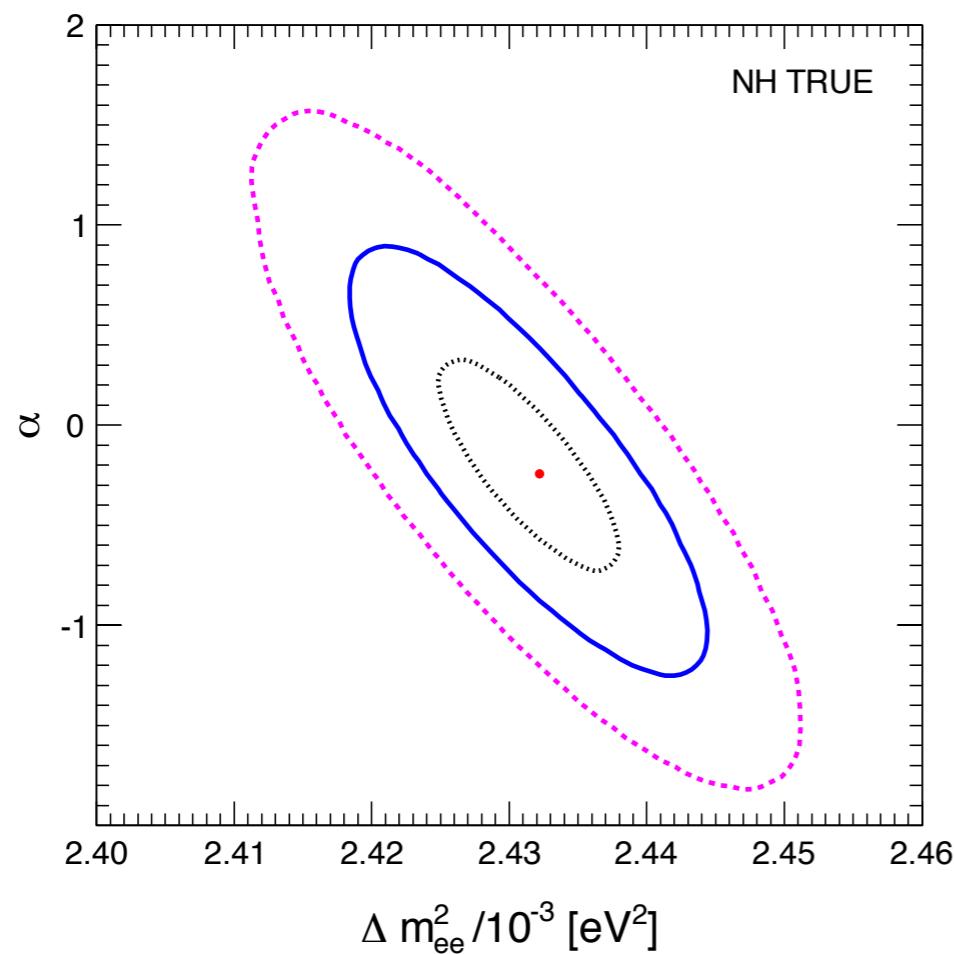
they could almost undo the low-energy spectral changes and adjust the fit



The “undecidable” case

$E \rightarrow E'$ so that $\alpha = \pm 1 \rightarrow \alpha = 0$

Hierarchy determination compromised



Conclusions

MBL reactor neutrino experiment purpose is to probe with unprecedented precision some oscillation parameters and the hierarchy. High precision required on both experimental and theoretical side

To this end we include

recoil effects in the cross section calculation (analytically)

matter effects in the oscillation probability (analytically)

multiple reactors effects by means of a damping factor (analytically)

Moreover, we treat α as a continuous parameter to solve issues related to the statistical interpretation of the data analysis

If energy scale errors are under control, with $\sim 10^5$ events

hierarchy determination at $\lesssim 2\sigma$

One order of magnitude better
measure of $(\delta m^2, \Delta m^2, \sin^2 \theta_{12})$